Applied Multivarate Analysis lab 2

1 Matrix algebra

1.1 Matrix operation

```
> A<-matrix(c(3,-1,0,-1,2,-1,0,-1,3),3,3)  # create 3 by 3 matrix
> A

     [,1] [,2] [,3]
[1,]    3   -1    0
[2,]   -1    2   -1
[3,]    0   -1    3

> B<-cbind(c(3,-2,4),c(1,3,7))  # bind together as columns
> B

     [,1] [,2]
[1,]    3    1
[2,]   -2    3
[3,]    4    7

> dim(B)  # dimension of the matrix (number of rows and columns)

[1] 3 2

> D<-matrix(1:6,nrow=3,ncol=2)  # create 3 by 2 matrix
> D

     [,1] [,2]
[1,]    1    4
[2,]    2    5
[3,]    3    6

> E<-matrix(1:6,nrow=3,ncol=2,byrow=T)  # matrix is filled by rows
> E

     [,1] [,2]
[1,]    1    2
[2,]    3    4
[3,]    5    6

> A%*%B  # matrix multiplication

     [,1] [,2]
[1,]   11    0
[2,]  -11   -2
[3,]   14   18
```

```
> B+D   # matrix addition

     [,1] [,2]
[1,]    4    5
[2,]    0    8
[3,]    7   13

> t(B)  # transpose

     [,1] [,2] [,3]
[1,]    3   -2    4
[2,]    1    3    7

> sum(diag(A))  # trace of matrix

[1] 8

> det(A)  # compute determinant

[1] 12

> prod(eigen(A)$values)  # compute determinant using eigenvalues

[1] 12

> solve(A)  # invert a matrix

           [,1] [,2]       [,3]
[1,] 0.41666667 0.25 0.08333333
[2,] 0.25000000 0.75 0.25000000
[3,] 0.08333333 0.25 0.41666667

> eigen(A)  # find the eigenvalues and eigenvectors of a square matrix

$values
[1] 4 3 1

$vectors
           [,1]          [,2]      [,3]
[1,]  0.5773503 -7.071068e-01 0.4082483
[2,] -0.5773503  4.710277e-16 0.8164966
[3,]  0.5773503  7.071068e-01 0.4082483

> p<-eigen(A)
> p$values

[1] 4 3 1

> p$vectors

           [,1]          [,2]      [,3]
[1,]  0.5773503 -7.071068e-01 0.4082483
[2,] -0.5773503  4.710277e-16 0.8164966
[3,]  0.5773503  7.071068e-01 0.4082483
```

1.2 Spectal Decomposition

```
> lambda<-matrix(0,3,3)  # create 3 by 3 matrix
> lambda

     [,1] [,2] [,3]
[1,]    0    0    0
[2,]    0    0    0
[3,]    0    0    0

> diag(lambda)<-p$values
> lambda

     [,1] [,2] [,3]
[1,]    4    0    0
[2,]    0    3    0
[3,]    0    0    1

> sqrtA<-p$vectors %*% sqrt(lambda)%*% t(p$vectors)
> sqrtA

            [,1]       [,2]        [,3]
[1,]  1.69935874 -0.3333333 -0.03269207
[2,] -0.33333333  1.3333333 -0.33333333
[3,] -0.03269207 -0.3333333  1.69935874

> invA<-p$vectors %*% solve(lambda)%*% t(p$vectors)
> invA

           [,1] [,2]       [,3]
[1,] 0.41666667 0.25 0.08333333
[2,] 0.25000000 0.75 0.25000000
[3,] 0.08333333 0.25 0.41666667

> sqrtA %*% sqrtA

              [,1] [,2]          [,3]
[1,]  3.000000e+00   -1 -1.804112e-16
[2,] -1.000000e+00    2 -1.000000e+00
[3,] -1.804112e-16   -1  3.000000e+00

> A %*% invA

              [,1]          [,2]          [,3]
[1,]  1.000000e+00  8.881784e-16  3.330669e-16
[2,]  5.412337e-16  1.000000e+00 -1.110223e-16
[3,] -2.775558e-16 -8.881784e-16  1.000000e+00
```

1.3 Access the rows, columns, or elements in the matrix

```
> X<-matrix(c(1,2,3,4,5,6,7,8,9),nrow=3,ncol=3)
> X

     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9

> Y<-matrix(c(9,8,7,6,5,4,3,2,1),nrow=3,ncol=3)
> Y

     [,1] [,2] [,3]
[1,]    9    6    3
[2,]    8    5    2
[3,]    7    4    1

> X[1,]   # get the 1st row

[1] 1 4 7

> X[,2]   # get the 2nd column

[1] 4 5 6

> X[-2,]  # delete the 2nd row

     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    3    6    9

> X[,-1]  # delete the 1st column

     [,1] [,2]
[1,]    4    7
[2,]    5    8
[3,]    6    9

> X>4

      [,1]  [,2] [,3]
[1,] FALSE FALSE TRUE
[2,] FALSE  TRUE TRUE
[3,] FALSE  TRUE TRUE

> X[X>4]

[1] 5 6 7 8 9
```

1.4 Append matrices together

```
> cbind(X,Y)

     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    1    4    7    9    6    3
[2,]    2    5    8    8    5    2
[3,]    3    6    9    7    4    1
```

```
> rbind(X,Y)

     [,1] [,2] [,3]
[1,]    1    4    7
[2,]    2    5    8
[3,]    3    6    9
[4,]    9    6    3
[5,]    8    5    2
[6,]    7    4    1
```